Simultaneous Borrowing and Saving in Microfinance

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Abstract

This paper provides a rationale for one of the widely practised mechanisms by MFIs – simultaneous borrowing and saving. Unlike the existing literature, our explanation does not involve any behavioral aspect. We study a dynamic relationship between a benevolent MFI and a strategic borrower. The optimum contract involves simultaneous borrowing and saving – at each date, the MFI provides a small loan, the borrower invests that in a production technology, and saves the net return with the MFI. These help her to accumulate a lumpsum amount and "graduate" to an improved lifetime utility which is not achievable when only credit is provided. Over time, as her savings increase, her incentive to repay increases. The optimal loan scheme is weakly progressive i.e. weakly increasing over time. We also provide a sufficient condition for the non-existence of any incentive compatible contract in our framework.

JEL Classification Number: O12, O16, D86, G21.

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1 Introduction

Simultaneous borrowing and saving have been practised by many microfinance institutions (hereafter MFIs), e.g. Grameen II, and FINCA Nicaragua. However, it may seem counter-intuitive since, with money being fungible, it would make more sense to just save the net amount instead. In contrast to the existing literature, we provide a rationale for this practice that does not involve any behavioral aspect. In particular, we develop a theoretical model where the MFI provides a (locked-in) savings service along with credit,[1] which increases a poor borrower's lifetime utility beyond the level achievable when only credit is provided. Interestingly, the borrower's welfare-maximizing loan sequence is progressive in nature – increasing over time contingent on successful repayment – a practice that is common to almost all the MFIs (Grameen II, FINCA Nicaragua for example). Thus, in addition to providing a rationale for simultaneous borrowing and saving, this paper contributes to the literature by studying the impact of (locked-in) savings on loan size, as well as developing an explanation of progressive lending, two other institutional aspects that have attracted relatively less attention in the literature.[2]

Formally, we study a dynamic relationship between a borrower with no endowment and a benevolent MFI whose objective is to maximise the borrower's lifetime utility subject to a break-even condition. The borrower has access to a project that requires a fixed initial investment ($\bar{S}$ say). We may think of this $\bar{S}$ as the amount required for renting a formal storefront with a minimum scale, acquiring an asset, hiring an employee, etc. (see Banerjee et al. (2019)). When a poor borrower starts investing in this non-convex technology we say that she has *graduated*. The problem is that she is subject to an *ex post* moral hazard problem in that she does not repay whenever she has an incentive to do so and it is *not* possible for the MFI to incentivise her to repay once she graduates. Therefore, the first-best where the MFI lends the required amount so that the borrower can graduate immediately and repays after that, is not achievable. Hence, the MFI has to design a contract such that the borrower does not have any incentive to default and her utility is maximised. However, we show that the MFI can solve this incentive problem by providing conditional access to a productive technology which does not require any initial fixed investment and is less productive than the non-convex technology discussed earlier. In case of default, the borrower loses access to this technology.

We first characterize the optimal contract under this scenario where the borrower has limited access to technologies.[3] The optimal contract involves both savings and credit – at each period, the MFI provides a small amount of loan which the borrower invests in the technology, access to which is given by the MFI. After she gets the return from that investment, she decides whether to repay or not. In case she repays, she also saves a part of the net return with the MFI and gets another small loan, with the same conditions. This process continues until her total savings become large enough ($\bar{S}$) to graduate. In case of default, the MFI terminates the contract and confiscates the borrower's savings with it till date. The termination of the contract entails that the borrower loses access to all future loans, as well as the access to technology which was provided by the MFI.

[1] Many MFIs provide such a service. It has been argued that "collateralizing mandatory savings could offer a win-win solution for both lender and borrower by providing the MFP" (where MFP stands for Microfinance Providers) "with security while at the same time building the asset base of the client." (Aslam and Azmat (2012)). For more evidence see Appendix B of the Working Paper.

[2] While there are quite a few papers which address progressive lending, most of them involve default along the equilibrium path, however, we observe near perfect repayment rate in microfinance. For more examples of MFIs which practise progressive lending and compulsory savings see the Working Paper.

[3] We shortly discuss a scenario when the borrower has access to all the available technologies in the economy. In Section 4, we show that borrower's limited access to technology is a necessary condition for the existence of any incentive compatible contract.

Since graduation is welfare improving, after default, the borrower saves the amount with which she defaults and graduates as soon as that becomes $\bar{S}$.

The optimal contract of the MFI enables the borrower to graduate[4] *as soon as possible*: Along the equilibrium path, the MFI terminates the contract as soon as the borrower's total savings (along with interest) become $\bar{S}$. The borrower's savings increase with the net return at any date. So, the MFI lends in such a way that the net return is maximised, which requires lending the efficient amount[5] if that is incentive compatible, or the maximum amount that is. We further find that to reduce the time required to accumulate $\bar{S}$, the borrower saves the entire net return with the MFI.

The optimum loan scheme is weakly progressive, i.e. nondecreasing over time. Intuitively, as time passes, the amount saved with the MFI increases, so the time remaining to graduate decreases, which implies that the present discounted payoff from graduation increases over time. Moreover, we assume that in case of default, the MFI confiscates her entire savings till date and she loses access to the technology provided by the MFI. Thus to graduate, she saves the amount with which she defaults. Therefore, the incentive compatibility constraint gets relaxed if the loan amount decreases (or remains constant) over time. The MFI, being benevolent, hence would increase the loan amount whenever that is lower than the efficient amount. We further find that, depending on the productivity of the MFI's technology and the fixed initial investment required to graduate $\bar{S}$, the efficient amount may become incentive compatible from the very beginning, or after a few periods towards the end, or never. Accordingly, the optimum loan scheme can be of three types – (i) constant – the optimum loan amount remains constant at the efficient level, (ii) progressive with a cap – it initially increases and then remains constant at the efficient level, and (iii) strictly progressive – it increases at all periods. In reality, most MFIs practise progressive lending with a cap.[6] In our framework, the optimal loan scheme is such when the production technology available before graduation is not very productive and the fixed initial investment required to graduate is moderate.[7]

We end the paper with an impossibility result – if the borrower has access to all the technologies available in the economy, then there does not exist any incentive compatible contract. We show that, given any contract, there exists a time period at which the borrower's lifetime utility from default is strictly higher than that from repayment, and she certainly, defaults at that time. This is an extension of the well-known non-existence result of Bulow and Rogoff (1989) and Rosenthal (1991) to our framework.

Now, we briefly discuss the related literature. Simultaneous borrowing and saving have mostly been explained behaviorally – Laibson et al. (2003), Baland et al. (2011), and Basu (2016) for example. In particular, Laibson et al. (2003) assume that agents are present biased to explain 'debt-puzzle' – they borrow aggressively on credit cards, and simultaneously save for retirement. Baland et al. (2011) explain how poor people save and borrow simultaneously to pretend to be poor so that they do not have to lend to their poor relatives. The paper closest to ours is Basu (2016). It develops a three-period model where a sophisticated present biased agent has an opportunity to invest at period 1. Without any commitment, due to present bias, he does not invest when the time comes. To make him invest, his period zero self simultaneously borrows and saves in a risky

[4] Banerjee et al. (2019) find that the MFIs help the households to escape from the fixed-cost-driven poverty trap and move into a more productive technology.

[5] The efficient amount is that amount for which the net return is maximised (formal definition later).

[6] Bandhan, Grameen, BRAC, to name a few. For more examples see Appendix B of the Working Paper.

[7] This conforms with reality as MFIs fund small businesses and Banerjee et al. (2015) in their famous six-country study find that increase in utility from microfinance is "modestly positive". Further, while it is difficult to estimate the required fixed initial investment to start a more productive, non-convex technology, one of the first papers to estimate that is Banerjee et al. (2019). Their estimated fixed cost is Rs. 7,900.

asset such that his wealth at that period remains the same. But the expected wealth at period 2 decreases. This makes the risk-averse agent invest at period 1. Our explanation does not rely on any behavioral aspect, instead benevolent MFIs optimally choose to provide credit and savings service simultaneously in order to improve the welfare of the poor agents who are subject to *ex post* moral hazard.

There are a few papers that address progressive lending in microfinance. In a two-period model Armendàriz and Morduch (2000) show how a *strategic* borrower's incentive to repay in the first period increases with an increase in the size of the second period loan. The equilibrium involves default in the second period though. In Ghosh and Ray (2016) progressive lending helps in weeding out the borrowers who never repay. Egli (2004) shows that progressive lending may fail to identify a "bad" type, since a bad borrower may camouflage herself as a "good" borrower (who always repays) in order to get a higher amount of loan later on which she defaults with certainty. Shapiro (2015) examines a framework with uncertainty over borrowers' discount rates. He shows that even in the efficient equilibrium almost all the borrowers default. Note that all these papers involve default along the equilibrium path. In contrast, our paper provides an explanation of progressive lending with no default along the equilibrium path as we observe almost no default on MFI-loans.

2 The Basic Framework – Payoffs, Technologies and Graduation

In an infinite horizon, discrete time framework, we study a dynamic relationship between a benevolent MFI that can, simultaneously, provide credit and savings services, subject to a zero profit condition, and a poor borrower who is subject to an *ex post* moral hazard problem in that she does not repay whenever she has an incentive to do so. We also assume that the borrower is protected by limited liability constraint.

The borrower has access to a non-convex technology $\langle V, \bar{S} \rangle$, where $\bar{S}(>0)$ is the required fixed initial investment, and V is the present discounted value of lifetime utility from investing in that technology (gross of $\bar{S}$). In this economy, to transfer wealth from one period to another, two other technologies which do not need any minimum initial investments are available – a deterministic neoclassical production technology $f(\cdot)$ and a savings technology. The net interest rate on savings is r. We assume the interest rate r and the common future discount factor δ are related as follows:[8]

Assumption 1. $\delta = \dfrac{1}{1+r}$.

In the neoclassical production technology $f(\cdot)$, if an amount k is invested at period t, then it produces $f(k)$ in the next period. Further, $f(\cdot)$ satisfies the usual assumptions:

Assumption 2. $f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0,\ \lim_{k \to 0} f'(k) = \infty$ *and* $\lim_{k \to \infty} f'(k) = 0$.

We denote the efficient scale of investment by k^e which solves $\underset{k}{\operatorname{argmax}}\ [f(k) - (1+r)k]$.

We assume that the borrower has a linear utility function. We also assume that the net gain from investing in the $\langle V, \bar{S} \rangle$ technology exceeds the present discounted net payoff from running the $f(\cdot)$ technology at its efficient level:

Assumption 3. $V - \bar{S} > \sum_{t=1}^{\infty} \delta^t [f(k^e) - (1+r)k^e] = \dfrac{\delta}{1-\delta}[f(k^e) - (1+r)k^e]$,

[8]We make this assumption to abstract from time trends created purely from time preference.

where recall $\bar{S}(>0)$ is the required fixed initial investment, and V is the present discounted value of lifetime utility from investing in that technology (gross of $\bar{S}$). For this reason, once the borrower starts investing in $\langle V, \bar{S} \rangle$ technology, we say that she has *graduated*.

Observe the first-best is to graduate immediately but that is not achievable as the borrower's endowment is zero, so she cannot start the project on her own. The MFI could provide adequate credit $\bar{S}$ so that the borrower could graduate immediately. However, that is also not possible as the borrower would default and the MFI would make a loss with certainty.[9] In fact, the existence of any incentive compatible contract depends on the extent of borrower's access to various technologies before graduation. As we shall later find that such contracts exist only when she has limited access to technology.[10]

3 Simultaneous Borrowing and Saving

In this section, we characterize the optimal contract(s) when the borrower does not have access to the $f(\cdot)$ technology and has access to a savings technology and $\langle V, \bar{S} \rangle$ technology, on her own. We find that the optimum contract involves simultaneous borrowing and saving – at each period, the borrower receives a loan, invests in the $f(\cdot)$ technology, and after repayment, saves the rest with the MFI. We also find that the optimum loan amount is progressive in that it (weakly) increases over time. Further, the optimum contracts end at a finite date and the borrower graduates immediately after the successful completion of the contract.

3.1 Contracts and Timeline

We consider an infinite horizon, discrete time framework where $t \geq 0$. At $t = 0$, the MFI announces a contract $\left\langle \{k_t(w_t, s_t)\}_{t=0}^{T_M - 1}, T_M \right\rangle$ where T_M is the 'successful' termination date of the contract and k_t is the loan amount at any period $t \in \{0, ..., T_{M-1}\}$. The loan amount at each period t is a function of the borrower's savings with the MFI till date w_t and her own independent savings s_t.

At any period $t \in \{1, ..., T_M\}$, the borrower decides whether to repay the last period's loan (along with interest) $(1 + r)k_{t-1}$ or not.[11] If she repays, then she also decides how much to save with the MFI and how much on her own. The savings with the MFI is assumed to be locked,[12] that is, the borrower can choose the savings rate with the MFI, but once saved, she cannot withdraw her savings with the MFI till the successful termination of the contract. If the borrower defaults at any period t ($\in \{1, ..., T_M\}$), then she loses access to all future loans, access to the $f(\cdot)$ technology, and all of her savings with the MFI till date. We call this 'unsuccessful' termination of the contract. After the termination of the contract, either successfully or unsuccessfully, the borrower operates on her own and chooses the graduation date and the savings rate from the termination date till the

[9] This is a direct consequence of Bulow and Rogoff (1989) and Rosenthal (1991).

[10] In the next section, we characterize one particular case where the borrower has limited access to technologies – she does not have access to the $f(\cdot)$ technology on her own. The analyses of other two cases – (a) she does not have access to the savings technology and (b) she does not have access to both $f(\cdot)$ and savings technologies – can be done analogously. However, those are beyond the scope of this paper. Section 4 addresses this non-existence problem.

[11] Here, we assume that at any period $t \in \{1, ..., T_M\}$, the borrower is supposed to repay the entire amount she had borrowed at period $t - 1$ (along with interest). In Remark 1, we discuss that it is without loss of generality.

[12] Here, we only consider the *locked-in* savings option. However, even if the borrower could have chosen between *locked-in* and *no locked-in* savings options, she would have, optimally, chosen the former. We discuss this in greater details in Remark 2.

graduation date.[13]

The timeline is as follows. At $t = 0$, the MFI announces a contract $\left\langle \{k_t(w_t, s_t)\}_{t=0}^{T_M-1}, T_M \right\rangle$. The borrower either accepts or rejects this contract, with the game ending in case she rejects (as the borrower's endowment is zero). If the borrower accepts then she gets $k_0(0,0)$ amount of loan and she invests that in the $f(\cdot)$ technology. The continuation game at any period $t \in \{1, .., T_M - 1\}$ is as follows:

Stage 1: $f(k_{t-1})$ is produced. The borrower decides whether to repay $(1+r)k_{t-1}$ or not. If she does not repay, then the contract gets terminated, unsuccessfully. The borrower loses access to all future loans, access to $f(\cdot)$ technology and also, all of her savings with the MFI till date.

Stage 2: If she repays, then after the repayment, money in her hand comprises of net return $f(k_{t-1}) - (1+r)k_{t-1}$ and her own independent savings $(1+r)s_{t-1}$ (if any). She saves a fraction α_t of that with the MFI, a fraction β_t on her own, and consumes the rest. We shall consider $0 \leq \alpha_t, \beta_t$ and given limited liability, $\alpha_t + \beta_t \leq 1$.

Stage 3: The MFI observes the borrower's total savings with it till date, i.e $w_t = \alpha_t[f(k_{t-1}) - (1+r)k_{t-1} + (1+r)s_{t-1}] + (1+r)w_{t-1}$, and also her own independent savings i.e. $s_t = \beta_t[f(k_{t-1}) - (1+r)k_{t-1} + (1+r)s_{t-1}]$, and lends $k_t(w_t, s_t)$. The borrower invests this amount $k_t(w_t, s_t)$ in the $f(\cdot)$ technology and the game moves to the next period.

At period T_M, $f(k_{T_M-1})$ is produced. The borrower decides whether to repay $(1+r)k_{T_M-1}$ or not. If she repays, the contract gets terminated successfully and she gets back all of her savings with the MFI till date, i.e. $(1+r)w_{T_M-1}$ whereas, if she does not repay, then the contract gets terminated, unsuccessfully, and the MFI confiscates her entire savings with it.

After the termination of the contract (either successfully or unsuccessfully), the borrower chooses graduation date and savings rate at each period till that date.

We solve for the subgame perfect Nash equilibrium (SPNE) with endogenous end date.

3.2 Characterization of The Optimal Contracts: Simultaneous Borrowing and Saving

We, first, consider the problem of the borrower after the successful termination of the contract at T_M. Money in her hand at that time includes net return after repayment, total deposit with the MFI till date (along with interest) and her own independent savings till date. Let us denote money in hand by ω. Recall, she has access to the savings and $\langle V, \bar{S} \rangle$ technology but not to $f(\cdot)$ technology on her own. Hence, at T_M, her problem is to choose a graduation date T and a savings rate till date $\beta_t^A \in [0,1]$ $\forall t \in \{T_M, ..., T-1\}$ to maximize the present discounted value of her lifetime utility subject to GC, the graduation constraint, which ensures that at the time of graduation she has at

[13] For the ease of exposition, here, we assume that the borrower graduates at a finite date T, and $T_M \leq T$, so T_M is also finite. In the Working Paper, we show that at the optimum, the borrower, indeed, graduates at a finite date which is a direct consequence of our assumption that graduation is welfare improving (Assumption 3).

least $\bar{S}$ amount of savings, i.e. her problem is to

$$\mathcal{U}^A(\omega) \equiv \underset{\langle\{\{\beta_t^A\}_{t=T_M}^{T-1}, T\rangle}{\text{Maximise}} (1-\beta_{T_M}^A)\omega + \sum_{t=T_M+1}^{T-1} \delta^{t-T_M}(1-\beta_t^A)[(1+r)s_{t-1}^A] + \delta^{T-T_M}\left[s_T^A - \bar{S} + V\right]$$

where $s_{T_M}^A \equiv \beta_{T_M}^A \omega$, and $\forall t \in \{T_M+1, ..., T-1\}$ $s_t^A \equiv \beta_t^A[(1+r)s_{t-1}^A]$, and $s_T^A \equiv (1+r)s_{T-1}^A$,
Subject to (i) $\beta_t^A \in [0,1]$ $\forall t \in \{T_M, ..., T-1\}$, and (ii) Graduation Constraint (GC): $s_T^A \geq \bar{S}$.

The objective function has two components – borrower's utility from consumption from period T_M till $T-1$. And, the borrower's utility from period T onwards. At period T, the borrower invests $\bar{S}$ to graduate, the present discounted value of lifetime utility from which is V, and consumes the rest. As mentioned above, the graduation constraint GC ensures that at period T, she has enough savings to invest $\bar{S}$. The solution to this problem is denoted by $\mathcal{U}^A(\omega)$. Similarly, consider the problem of the borrower after she defaults at period τ. Money in her hand at that time is $f(k_{\tau-1})$ and her own independent savings till date $(1+r)s_{\tau-1}$. Her problem is to choose a graduation date $T^A(\tau)$ and the savings rate $\beta_t^A(\tau) \in [0,1]$ $\forall t \in \{\tau, ..., T^A(\tau)-1\}$. Since the problem is very similar to the problem depicted above, we skip it here. The optimal choices, after termination of a contract, depend only on the money in the borrower's hand at the time of termination. We denote the optimal solution by $\mathcal{U}^A(\omega)$ in both the cases, where ω is the money in hand at the time of termination. We find that

The next lemma characterizes the optimal choices after the termination of a contract successfully (or unsuccessfully). We find that the borrower optimally chooses to graduate as soon as possible. The proof is straightforward and can be found in the Working Paper.

Lemma 1. *After the successful (or unsuccessful) termination of the contract at T_M (or at τ), at the optimum, the borrower graduates as soon as possible –*

i. if money in her hand, at the termination date T_M (or τ), is no less than $\bar{S}$, then she graduates immediately,

ii. otherwise, (a) she graduates as soon as her savings becomes $\bar{S}$, i.e. $s_{T^*-1}^A < \bar{S} \leq s_{T^*}^A$ $\big($or $s_{T^{A^*}(\tau)-1}^A < \bar{S} \leq s_{T^{A^*}(\tau)}^A\big)$, and (b) at each period till then, she saves as much as she can, i.e. $\beta_t^{A^*} = 1$ $\forall t \in \{T_M, ..., T^*-1\}$ $\big($or $\beta_t^{A^*}(\tau) = 1$ $\forall t \in \{\tau, ..., T^{A^*}-1\}\big)$.

Intuitively, given our assumptions that the borrower has a linear utility function, and discounts future as the inverse of the interest rate on savings (Assumption 1), she is indifferent between consuming an amount now and saving and consuming that amount (along with interest) later. However, as graduation is welfare improving (Assumption 3), and beyond $\bar{S}$, V is independent of the wealth with which she graduates, the present discounted value of her lifetime utility is the maximum when the time required to graduate is the minimum. Hence, if the money in her hand at the time of termination of the contract is no less than $\bar{S}$, she graduates immediately, otherwise, she saves the maximum amount possible at each period and graduates as soon as her total savings become no less than $\bar{S}$.

Now, we consider the relationship between the MFI and the borrower. The problem of the MFI is to choose $\langle\{k_t(w_t, s_t)\}_{t=0}^{T_M-1}, T_M\rangle$[14] to maximize the borrower's present discounted value of lifetime utility subject to the DIC constraints which ensure that she repays always, formally, the

[14]For the brevity of notation, we shall denote $k_t(w_t, s_t)$ by k_t, except when explicit reference to w_t and s_t is required.

problem is to

$$\underset{\langle\{k_t\}_{t=0}^{T_M-1},T_M\rangle}{\text{Maximise}} \sum_{t=1}^{T_M-1} \delta^t(1-\alpha_t-\beta_t)\big[f(k_{t-1})-(1+r)k_{t-1}+(1+r)s_{t-1}\big]$$
$$+\,\delta^{T_M}\mathcal{U}^A\Big(f(k_{T_M-1})-(1+r)k_{T_M-1}+(1+r)w_{T_M-1}+(1+r)s_{T_M-1}\Big)$$

where $\forall t \in \{1,...,T_M-1\}$

deposit with MFI: $w_t = \alpha_t\big[f(k_{t-1})-(1+r)k_{t-1}+(1+r)s_{t-1}\big]+(1+r)w_{t-1}$

her own savings: $s_t = \beta_t\big[f(k_{t-1})-(1+r)k_{t-1}+(1+r)s_{t-1}\big]$

Subject to the Dynamic Incentive Compatibility (DIC) constraints:

$$(i)\,\forall\tau \in \{1,...,T_M-1\} \quad \sum_{t=\tau}^{T_M-1} \delta^{t-\tau}(1-\alpha_t-\beta_t)\big[f(k_{t-1})-(1+r)k_{t-1}+(1+r)s_{t-1}\big]$$
$$+\,\delta^{T_M-\tau}\mathcal{U}^A\Big(f(k_{T_M-1})-(1+r)k_{T_M-1}+(1+r)w_{T_M-1}+(1+r)s_{T_M-1}\Big)$$
$$\geq \mathcal{U}^A\Big(f(k_{\tau-1})+(1+r)s_{\tau-1}\Big)$$

$$(ii)\,\mathcal{U}^A\Big(f(k_{T_M-1})-(1+r)k_{T_M-1}+(1+r)[w_{T_M-1}+s_{T_M-1}]\Big) \geq \mathcal{U}^A\Big(f(k_{T_M-1})+(1+r)s_{T_M-1}\Big).$$

Let us, first, discuss the objective function. At any period $t \in \{1,...,T_M-1\}$, the money in the borrower's hand is the net return from production after repayment and her own independent savings till date. She saves α_t part of that with the MFI, β_t part, independently, on her own, and consumes the rest. The first term of the objective function denotes the present discounted value of her utility from consumption at each period till T_M-1^{th} period. The second term denotes the present discounted value of her utility at the T_M^{th} period when after repayment, she gets back her entire savings (along with interest) and operates on her own from that period onwards. The money in her hand at that time comprises of the net return after repayment $f(k_{T_M-1})-(1+r)k_{T_M-1}$, her total savings with the MFI till date (along with interest) i.e. $(1+r)w_{T_M-1}$ and her own savings $(1+r)s_{T_M-1}$.

Let us, now, briefly explain the constraints. Consider any τ ($\in \{1,...,T_M-1\}$), the borrower's present discounted value of lifetime utility from repayment, i.e. the left hand side of the DIC can be explained as above – borrower gets utility from consumption till the T_M-1^{th} period starting from period τ and her utility at period T_M as stated above (both evaluated at period τ). The right hand side of the DIC is the borrower's utility from default at period τ, i.e. if she starts operating on her own with $f(k_\tau)$ and her own independent savings $(1+r)s_{\tau-1}$. Similarly, consider the DIC constraint at period T_M. The L.H.S. of this constraint represents the borrower's utility from repayment at period T_M and the R.H.S. represents her utility from default.

Before proceeding further let us introduce the following technical definition.

Definition 1. *Given a scheme* $\mathcal{L} \equiv \langle\{k_t\}_{t=0}^{T_M-1},\{\alpha_t\}_{t=1}^{T_M-1},\{\beta_t\}_{t=1}^{T_M-1},T_M,\{\beta_t^A\}_{t=T_M}^{T-1},T\rangle$*, let* $k_{I\tau}(\mathcal{L})$ *denote the maximum loan amount at* τ*, such that DIC at* τ *holds.*

Observe, the MFI would never lend more than the efficient amount[15] k^e as otherwise it is possible to increase the borrower's utility, by decreasing the loan amount which would increase the net return from production, without violating the DIC constraint of that period. Hence, at the optimum, the MFI lends k^e whenever that is incentive compatible, otherwise, it lends the maximum amount which is. We summarize this in the following observation.

[15]Recall the definition of k^e, it maximizes the net return.

Observation 1. *Let Assumption 2 hold and* $\mathcal{L} \equiv \langle \{k_t^*\}_{t=0}^{T_M-1}, \{\alpha_t\}_{t=1}^{T_M-1}, \{\beta_t\}_{t=1}^{T_M-1}, T_M, \{\beta_t^A\}_{t=T_M}^{T-1}, T\rangle$ *be any scheme where* k_t^* *is the optimum loan amount at any period* $t \in \{0, ..., T_M - 1\}$*, then* $k_t^* = min\{k^e, k_{It}(\mathcal{L})\}$*.*

Given this observation, consider the problem of the borrower, from period 1 till T_M, when she chooses to repay at all the periods (which is ensured by the DICs). The savings with the MFI is locked in, so after repayment at any period $t \in \{1, ..., T_{M-1}\}$, money in her hand is $f(k_{t-1}) - (1 + r)k_{t-1}$ plus her own independent savings $(1 + r)s_{t-1}$. She chooses a part (α_t) of it to save with the MFI and a part (β_t) to save on her own such that the present discounted value of her lifetime utility is the maximum. Hence, her problem is to

$$\underset{\langle \{\alpha_t\}_{t=1}^{T_M-1}, \{\beta_t\}_{t=1}^{T_M-1}\rangle}{\text{Maximise}} \sum_{t=1}^{T_M-1} \delta^t (1 - \alpha_t - \beta_t)\big[f(k_{t-1}) - (1 + r)k_{t-1} + (1 + r)s_{t-1}\big]$$
$$+ \mathcal{U}^A\Big(f(k_{T_M-1}) - (1 + r)k_{T_M-1} + (1 + r)w_{T_M-1} + (1 + r)s_{T_M-1}\Big)$$
$$\text{Subject to} \quad \forall t \in \{1, ..., T_M - 1\} \quad \alpha_t, \beta_t \geq 0 \text{ and } \alpha_t + \beta_t \leq 1.$$

In the next lemma, we characterize the borrower's optimal choices of the savings rates – the part of money in her hand she saves with the MFI, i.e. α_t and that she saves on her own, i.e. β_t. The proof can be found in the Appendix.[16]

Lemma 2. *Let Assumptions 1, 2, 3 hold, and* $\langle \{k_t^*\}_{t=0}^{T_M-1}, \{\alpha_t^*\}_{t=1}^{T_M-1}, \{\beta_t^*\}_{t=1}^{T_M-1}, T_M, \{\beta_t^{A^*}\}_{t=T_M}^{T^*-1}, T^*\rangle$ *be any scheme where* $\beta_t^{A^*}$, T^*, k_t^*, α_t^* *and* β_t^* *are optimally chosen. Then, at any period* $t \in \{1, ..., T_M - 1\}$*, the borrower saves the entire money in her hand with the MFI, i.e.* $\alpha_t^* = 1$ *and independently saves nothing, i.e.* $\beta_t^* = 0$*.*

The intuition is very similar to that when she operates on her own. Given our assumptions that the borrower has a linear utility function, discounts future as the inverse of the interest rate on savings (Assumption 1), graduation is welfare improving (Assumption 3), and beyond $\bar{S}$, V is independent of the wealth with which she graduates, maximizing borrower's utility boils down to minimizing the time required to graduate. Given Lemma 1, it implies that her objective is to maximize the amount with which her relationship with the MFI gets, successfully, terminated, i.e. her objective is to maximize the money in her hand at T_M. So, she maximizes her savings at each period, i.e. at any $t \in \{1, ..., T_M - 1\}$, she, optimally, chooses α_t and β_t, such that $\alpha_t^* + \beta_t^* = 1$.

Now, consider the choices of α_t^* and β_t^*, separately. For that consider their effects on the money in the hand of the borrower at T_M. Observe, *given* a loan sequence $\{k_t\}_{t=0}^{T_M-1}$, the choices of $\{\alpha_t\}_{t=1}^{T_M-1}$ and $\{\beta_t\}_{t=1}^{T_M-1}$ do not affect it. However, the DICs get affected by those choices – at any period $t \in \{1, ..., T_M - 1\}$, a higher amount of loan becomes incentive compatible with an increase in α_t and a decrease in β_t. Hence, given Observation 1, the net return from production, and hence, savings at each period, (weakly) increases with an increase in α_t and a decrease in β_t. Thus, at each period $t \in \{1, ..., T_M - 1\}$, the borrower chooses to save the entire money in her hand with the MFI, i.e. $\alpha_t^* = 1$, which implies she saves nothing independently, i.e. $\beta_t^* = 0$.

Finally, we characterize the successful termination date of an optimum contract. For that, we

[16] To prove this, we assume that if a contract provides (weakly) higher utility than another contract, then the former is the optimum.

consider the MFI's problem, given the optimal choices we have characterized, till now. It is to

$$\underset{\langle\{k_t\}_{t=0}^{T_M-1},T_M\rangle}{\text{Maximise}} \quad \delta^{T^*}\mathcal{U}^A\Big(\sum_{t=1}^{T_M}(1+r)^{T_M-t}[f(k_{t-1})-(1+r)k_{t-1}]\Big)$$

Subject to DIC constraints: $\forall\tau\in\{1,...,T_M\}$

$$\delta^{T^*-\tau}\mathcal{U}^A\Big(\sum_{t=1}^{T_M}(1+r)^{T_M-t}[f(k_{t-1})-(1+r)k_{t-1}]\Big)\geq\mathcal{U}^A(f(k_{\tau-1})).$$

In the next lemma, we show that that the MFI chooses to terminate the contract successfully, as soon as money in the borrower's hand (at the time of successful termination of the contract) becomes no less than $\bar{S}$. The proof can be found in the Appendix.

Lemma 3. *Let Assumptions 1, 2, 3 hold, and* $\langle\{k_t^*\}_{t=0}^{T_M^*-1},\{\alpha_t^*\}_{t=1}^{T_M^*-1},\{\beta_t^*\}_{t=1}^{T_M^*-1},T_M^*,\{\{\beta_t^{A^*}\}_{t=T_M^*}^{T^*-1},T^*\rangle$ *be any optimum scheme, then the MFI chooses* T_M^* *in such a way that*

$$S^*_{T^*_M}\equiv\sum_{t=1}^{T^*_M}(1+r)^{T^*_M-t}[f(k^*_{t-1})-(1+r)k^*_{t-1}]\geq\bar{S}>\sum_{t=1}^{T^*_M-1}(1+r)^{T^*_M-1-t}[f(k^*_{t-1})-(1+r)k^*_{t-1}]\equiv S^*_{T^*_M-1}.$$

Intuitively, as graduation is welfare improving (Assumption 3), the objective of the benevolent MFI is to minimize the time required to graduate, subject to DIC constraints. For that, first, we argue that the MFI must lend till the time borrower accumulates enough wealth to graduate, and it should also not delay the date of graduation after that accumulated wealth becomes no less than $\bar{S}$. Then, we argue that choosing the successful termination date in such a manner satisfies the DIC constraints at all the dates.

To argue that the MFI must lend till the time she graduates, recall that the $f(\cdot)$ technology is more productive than the savings technology when the investment amount is less than the efficient amount k^e. And, we have observed that the MFI enables the borrower to invest that amount whenever that is incentive compatible, otherwise the maximum amount which is. So, the time required to graduate is the minimum, when she invests in such a way till the time of graduation. Further, due to our assumption that beyond $\bar{S}$, V is independent of the wealth with which she graduates, the MFI, optimally, terminates the contract as soon as her accumulated wealth becomes no less than $\bar{S}$. Given Lemma 1, which implies that the borrower graduates, immediately at T_M^*.

We argue that terminating a contract in such a way satisfies the DIC constraints at all periods. This is because if the borrower defaults, she loses access to the more productive $f(\cdot)$ technology. To retain access to that technology she repays. Observe, if the borrower had only lost access to $f(\cdot)$ technology (along with future loans), she would have defaulted at the termination date of the contract, and that would have unraveled the entire contract. But, the borrower also loses her entire savings with the MFI, in order to get back that, the borrower chooses to repay even at the termination date. In fact, as time passes, the borrower becomes willing to repay a higher amount of loans. We explore this more in the next subsection where we characterizes the dynamics of the optimal contract.

We end this subsection by summarizing the full characterization of the optimum contract between the MFI and the borrower: The optimum contract involves simultaneous borrowing and saving – at each period, the borrower receives the efficient amount k^e as loan whenever that is incentive compatible, otherwise the maximum amount which is. She invests that in the $f(\cdot)$ technology and saves the entire net return with the MFI. The optimum contract gets terminated as soon as the borrower accumulates enough wealth to graduate, that is $\bar{S}$.

Proposition 1. (Simultaneous Borrowing and Saving) *Let Assumptions 1, 2, and 3 hold. Consider any optimum scheme* $\langle \{k_t^*\}_{t=0}^{T_M^*-1}, \{\alpha_t^*\}_{t=1}^{T_M^*-1}, \{\beta_t^*\}_{t=1}^{T_M^*-1}, T_M^*, \{\beta_t^{A*}\}_{t=T_M^*}^{T^*-1}, T^* \rangle$. *The borrower borrows from and saves with the MFI simultaneously:*

(a) The MFI lends the efficient amount whenever that is DIC, otherwise, it lends the maximum amount which is. Formally $k_t^* = \min\{k_{It}, k^e\}$ *for all* $t \in \{0, ..., T_M^*\}$.

(b) The borrower saves the maximum possible amount with the MFI, formally $\alpha_t^* = 1$ *and* $\beta_t^* = 0$ $\forall t \in \{1, ..., T_M^* - 1\}$.

(c) The borrower graduates at a finite date, as soon as her total wealth becomes no less than $\bar{S}$.

3.3 The Time Path of The Optimal Loan Scheme

We next characterize the time path of the optimal loan scheme. We find that it is progressive in nature, i.e. (weakly) increasing over time. Depending on the productivity of $f(\cdot)$ technology, and the required initial investment to graduate, i.e. $\bar{S}$, the optimum loan scheme could be one of the following three types – (a) *constant over time* – remains constant at the efficient amount k^e, (b) *progressive with a cap* – increases till the optimum loan amount becomes k^e, after which it remains constant over time, and (c) *strictly progressive* – increases at all t. To characterize the parametric condition under which each of these is optimal, we introduce the following two definitions.

Definition 2. *We say that the* $f(\cdot)$ *technology is very productive when* $f(k^e) \geq (1 + r)(2 + r)$, *otherwise we say that* $f(\cdot)$ *is not very productive.*

Definition 3. *We say that the required initial investment* $\bar{S}$ *is*

(a) large when $\dfrac{f(k^e)[f(k^e) - (1 + r)k^e]}{(1 + r)[f(k^e) - (1 + r)k^e] - rf(k^e)} \leq \bar{S}$,

(b) moderate when $(1 + r)f(k^e) \leq \bar{S} < \dfrac{f(k^e)[f(k^e) - (1 + r)k^e]}{(1 + r)[f(k^e) - (1 + r)k^e] - rf(k^e)}$, *and*

(c) small when $\bar{S} \leq f(k^e)$.

The next proposition states the main result of this subsection, the proof of which can be found in the Working Paper.

Proposition 2. *(The Dynamics of the Optimal Loan Scheme): Let Assumptions 1, 2, and 3 hold.*

A. The optimal loan scheme is weakly progressive over time.

B. If the $f(\cdot)$ *technology is very productive (as in Definition 2), then the optimal loan scheme is*[17]

(i) "strictly progressive" if the required initial investment in $\langle V, \bar{S} \rangle$ *is "small",*

(ii) "constant" if the required initial investment in $\langle V, \bar{S} \rangle$ *is "large".*

C. If the $f(\cdot)$ *technology is not very productive (see Definition 2), then the optimal loan scheme is*

(i) "strictly progressive" if the required initial investment in $\langle V, \bar{S} \rangle$ *is "small",*

[17] In this discrete version, we can only find the sufficient conditions for the optimum loan scheme to be strictly progressive or progressive with a cap. We find both necessary and sufficient conditions in the continuous version Dasgupta et al. (2020).

(ii) "progressive with a cap" if the required initial investment in $\langle V, \bar{S} \rangle$ is "moderate",

(iii) "constant" if the required initial investment in $\langle V, \bar{S} \rangle$ is "large".

Let us now discuss the intuition behind these results. Due to Lemma 2, in particular, $\alpha_t^* = 1$ at all t till graduation, the deposit with the MFI increases over time. Thus, with the passage of time, on the one hand, the borrower's savings increase, so that the loss from default increases, whereas on the other hand, the graduation date T_M^* gets closer, so the present discounted value of lifetime utility from repayment increases. This ensures that the DIC constraints get relaxed over time. Given Observation 1, this implies that the optimal loan scheme is weakly progressive.

There can be three cases – the optimal loan scheme is (a) constant over time – remains constant at the efficient amount k^e, this happens when k^e becomes incentive compatible from the very beginning, (b) progressive with a cap – increases till the optimum loan amount becomes the efficient amount k^e, after which it remains constant over time, this happens when k^e is not incentive compatible initially, and later becomes incentive compatible, or (c) strictly progressive – keeps on increasing over time, this happens when k^e never becomes incentive compatible. Next, we discuss the intuition behind these three kinds of optimum loan schemes and the respective parametric conditions under which we observe each of them.

The optimum loan scheme is *strictly progressive* when $\bar{S}$ is small, as k^e is not incentive compatible at any $t < T_M^* - 1$. This is because in case of default with $f(k^e)$, she can graduate immediately and if she repays, she has to wait for at least one more period. Next, the optimum loan scheme is *constant* when $\bar{S}$ is large, as k^e is incentive compatible even at period 1 (when her incentive is the lowest). The intuition is as follows. Observe if she defaults at period 1, then she loses access to the $f(\cdot)$ technology (along with access to all future loans), but gains the amount she was supposed to repay i.e. k_0. When $\bar{S}$ is large, then multiple periods are required to accumulate that large $\bar{S}$. Hence, the loss from defaulting at period 1 – losing access to $f(\cdot)$ technology for all future dates is higher than the one time gain from default – the amount to be repaid. Finally, the optimum loan scheme is *progressive with a cap* when $f(\cdot)$ is not very productive and $\bar{S}$ is moderate, as k^e is not incentive compatible initially and it is towards the end. This is because, if she gets k^e from the very beginning then the time required to save $\bar{S}$ is not that much, hence the loss from losing access to technology is lower than the gain from not repaying k^e. And, towards the end, when the borrower has a positive amount of savings with the MFI which she loses in case of default, k^e becomes incentive compatible.

Remark 1. We have assumed that at any period $t \in \{1, ..., T_M - 1\}$, the borrower is supposed to repay the entire amount she had borrowed at period $t-1$ (along with interest). Observe, this is without loss of generality, in that any other contract which maximizes the borrower's lifetime utility subject to the MFI's no-loss condition would be identical.[18] The reason, evident from the DIC constraints, is as follows. Observe, in our optimum contract, at any period t, the entire amount produced remains with the MFI – the borrower repays $(1+r)k_{t-1}$ and then saves $f(k_{t-1}) - (1 + r)k_{t-1}$. The MFI returns her total savings, along with interest, at the successful termination of the contract. So, the contract does not depend on the timing of the repayment, alternatively, whether the borrower is repaying k_{t-1} at t^{th} period or at some $t+n^{th}$ period, where $t+n \in \{1, ..., T_M\}$ does not affect the DIC constraints (subject to limited liability). Therefore, the borrower gets the same amount of loan at each period. Her total savings and hence, her utility remains the same.

Remark 2. Suppose, at $t = 0$, the borrower may choose whether she wants to keep her savings with the MFI locked-in or not. If she chooses the former, then as we have addressed, she cannot

[18] This conforms with the finding of Field and Pande (2008) that repayment schedule does not affect the default rate. We thank an anonymous referee for pointing this out.

withdraw her savings with the MFI, till the successful termination of the contract. But, if she chooses the latter, she can withdraw (part of) her savings with the MFI anytime she wants to. Observe, this provision of withdrawal of her savings with the MFI, only affects the DIC constraints adversely which (weakly) reduces the optimum loan amounts. Hence, the time required to graduate (weakly) increases which (weakly) decreases the borrower's lifetime utility. Thus, the borrower is (weakly) better-off in choosing the 'locked-in' savings option. She is strictly better-off, when k^e is not incentive compatible at all dates when she chooses 'no locked-in' feature.

4 An Impossibility Result

We have analyzed a scenario where the borrower, on her own, has limited access to technologies. In particular, she does not have access to the neoclassical production technology $f(\cdot)$. We have shown that there exists an incentive compatible contract, and the optimum contract involves simultaneous borrowing and saving.

Now, we consider the case where the borrower, on her own, has access to all the technologies. We show that there does not exist any incentive compatible contract.[19] It is easy to see that when the MFI provides 'only credit' service, then there does not exist any incentive compatible loan contract. This is because the borrower would definitely default at a time when the amount to be repaid is the maximum. We then examine whether this result extends to the case where the MFI can also enforce "locked-in" savings. For that we consider the following candidate contract as, under this contract, the incentive for repayment is higher than that under any other contract (say for example, the borrower does not save with the MFI, or saves only a fraction of her net return at each period, or only a part of her savings with the MFI is confiscated etc.). Under the candidate contract, initially the MFI provides a small amount of loan. At each period, till the successful termination of the contract, if the borrower repays then she also has to save her entire net return with the MFI. The savings with the MFI is locked – at any period, if the borrower defaults then the entire amount saved till then would be confiscated. While it may seem that this candidate contract is incentive compatible – initially, the borrower repays to get bigger future loans, and later she repays to not lose her savings with the MFI – we show that it is not.

The formal proof can be found in the Working Paper, here we provide an intuitive overview of the argument. That the candidate contract is not incentive compatible implies that there must exist a time period such that the present discounted value of lifetime utility from default at that period is strictly higher than that from repayment. Next, we first identify such a time period and then prove its existence.

Consider the time period at which the amount to be repaid is strictly higher than the borrower's savings with the MFI till date. If there are multiple such time periods, then consider the time period which is the maximum of such periods, that is the time period closest to the graduation date when this inequality holds.[20] We argue that the borrower's utility from default at this time period is strictly higher than that from repayment. For that, observe, if the borrower defaults at any period, then she loses (i) her savings with the MFI till date and (ii) access to all future loans; and she gains the amount to be repaid at that period.

We argue, step by step, that her gain is strictly higher than the loss due to both (i) and (ii). Consider (i). By the choice of default time, the loss due to her savings being confiscated is strictly lower than the gain from not repaying the said amount (as the amount to be repaid is higher than

[19]This result is an extension of the non-existence result of Bulow and Rogoff (1989) and more generally Rosenthal (1991) to our framework where the borrower saves up to invest in a non-convex technology $\langle V, \bar{S} \rangle$.

[20]Here, we use the fact that the borrower graduates at a finite date, in case of repayment.

the savings). Consider (ii). One may ask, as the borrower also loses access to all future loans, what if at *some* future date, the borrower's wealth under the MFI contract becomes higher than her wealth when she operates on her own after defaulting at the said period? That is not possible for the following reasons. First, at the time of default, the borrower's total wealth in case of default is strictly higher than that in case of repayment. Second, by the choice of default period, at any future date, under the MFI contract, the loan amount is strictly lower than the borrower's savings with the MFI. Given these two consequences of the strategic choice of the default date, a recursive argument shows that at any future date (after defaulting at the said date), the borrower, on her own, would be able to invest the same amount the MFI would have enabled her to if she had repaid till then. Moreover, if she invests the same amount then her savings would be strictly higher than her savings under the MFI contract. Hence, her wealth would be strictly higher at any date after default.[21] Therefore, the borrower would either graduate faster in case of default or would graduate at the same date, but money in her hand at the time of graduation, in case of default would be strictly higher.

Next, we argue that the time period considered above exists. This is because at period 1, she is supposed to repay $(1+r)k_0(>0)$ and her savings with the MFI is zero.[22]

5 Conclusion

Many scholars, e.g. Armendàriz and Morduch (2005), Roodman (2009), among others argue that MFIs should provide not only credit but also other financial services like savings, insurance, etc. In fact, Rhyne (November 2, 2010) directly links the crisis that happened in the microfinance sector in the Indian state of Andhra Pradesh to lack of deposit collection. Many MFIs are broadening their initial focus on microcredit to include the provision of savings (and other) products (Karlan et al., 2014). In this paper, we develop a theoretical model where the MFI provides not just credit but also access to other services in particular a savings facility. This savings service coupled with the credit service help a poor borrower to accumulate a lumpsum amount which enables her to graduate. Thus we provide *one* explanation where savings coupled with credit indeed improve borrower's utility beyond the level achievable when only credit is provided.

Acknowledgement: We especially thank Dilip Mookherjee for numerous discussions, comments, questions, and suggestions. In this regard Dyotona Dasgupta acknowledges The World Bank for Robert S. McNamara fellowship and the Institute for Economic Development, Boston University for hosting her. We thank Shyamal Chowdhury, Bhaskar Dutta, Parikshit Ghosh, Dhritiman Gupta, Priyanka Kothari, Dushyant Kumar, Tridip Ray, and Arunava Sen for their comments. We also thank two anonymous referees for suggestions which have improved this paper. All errors are our own.

[21] She may optimally invest a different amount, in which case her wealth would be even higher.

[22] We thank an anonymous referee for suggesting us to explore the result of this section.

Appendix

Proof of Lemma 2. Following the proof of Lemma 1, it is easy to see that *given* a sequence of loan amounts $\{k_t\}_{t=1}^{T_M-1}$, at any $t \in \{1, ..., T_M - 1\}$, the borrower optimally chooses to save the entire money in her hand, i.e. $\alpha_t^* + \beta_t^* = 1$. Now, we argue that the borrower saves the entire money in her hand with the MFI, i.e. $\alpha_t^* = 1$ and $\beta_t^* = 0$ at all $t \in \{1, ..., T_M - 1\}$. We prove this by contradiction.

Let $\langle \{k_t^*\}_{t=0}^{T_M-1}, \{\alpha_t^*\}_{t=1}^{T_M-1}, \{\beta_t^*\}_{t=1}^{T_M-1}, T_M, \{\beta_t^{A^*}\}_{t=T_M}^{T^*-1}, T^* \rangle$ be a scheme where $\beta_t^{A^*}$, T^*, k_t^*, α_t^* and β_t^* are optimally chosen and contrary to the claim, $\exists \tau \in \{1, ..., T_M - 1\}$ such that $\alpha_\tau^* < 1$ and hence, from the argument above $\beta_\tau^* > 0$. Consider a new contract $\langle \{k_t^{'}\}_{t=0}^{T_M^{'}-1}, \{\alpha_t^{'}\}_{t=1}^{T_M^{'}-1}, \{\beta_t^{'}\}_{t=1}^{T_M^{'}-1}, T_M^{'}, \{\{\beta_t^{A^{'}}\}_{t=T_M^{'}}^{T^{'}-1}, T^* \rangle$ which is identical to the original contract except $\alpha_\tau^{'} = \alpha_\tau^* + \epsilon$, $\beta_\tau^{'} = \beta_\tau^* - \epsilon$ and

$$k_\tau^{'} = \begin{cases} k_\tau^* + \Delta & \text{if } k_\tau^* < k^e \\ k_\tau^* & \text{otherwise.} \end{cases}$$

Hence, under this new contract, at period τ, the borrower saves higher amount with the MFI. We argue that it is possible to choose a $\Delta > 0$, such that this new contract satisfies DIC constraints at all $t \in \{1, ..., T_M\}$ and also provides (weakly) higher utility than the original contract.

Under this new contract, at period $\tau + 1$, the borrower will be willing to repay higher than k_τ^* as her savings with the MFI and hence loss from default is higher. Hence, $\exists \Delta > 0$ such that DIC constraint at period $\tau + 1$ is satisfied. Also, observe for that Δ, under this new contract, the DIC constraint at any $t \in \{1, ...\tau\}$ is identical to that of the original contract, and the DIC constraint at any $t \in \{\tau + 2, ..., T_M\}$ gets relaxed. Thus, the new contract satisfies DIC constraints at all $t \in \{1, ..., T_M\}$.

Under this new contract, at period τ, the borrower gets (weakly) higher amount of loan, hence at period $\tau+1$, the net return is (weakly) higher. Therefore, the new contract provides her (weakly) higher utility and also satisfies DIC constraints at all t. Without loss of generality, we assume that if a contract provides (weakly) higher utility than another contract, then the former is the optimum. Hence, the original contract could not have been optimum. ■

Proof of Lemma 3. We, first, show by contradiction that

$$S_{T_M^*}^* \equiv \sum_{t=1}^{T_M^*} (1+r)^{T_M^*-t}[f(k_{t-1}^*) - (1+r)k_{t-1}^*] \geq \bar{S}.$$

Suppose not and $\langle \{k_t^*\}_{t=0}^{T_M^*-1}, \{\alpha_t^*\}_{t=1}^{T_M^*-1}, \{\beta_t^*\}_{t=1}^{T_M^*-1}, T_M^*, \{\{\beta_t^{A^*}\}_{t=T_M^*}^{T^*-1}, T^* \rangle$ is an optimum scheme and

$$S_{T_M^*}^* \equiv \sum_{t=1}^{T_M^*} (1+r)^{T_M^*-t}[f(k_{t-1}^*) - (1+r)k_{t-1}^*] < \bar{S}.$$

Given Lemma 1, this implies that the borrower graduates at $T^* > T_M^*$ where

$$\sum_{t=1}^{T_M^*} (1+r)^{T^*-t}[f(k_{t-1}^*) - (1+r)k_{t-1}^*] \geq \bar{S} > \sum_{t=1}^{T_M^*} (1+r)^{T^*-1-t}[f(k_{t-1}^*) - (1+r)k_{t-1}^*].$$

The present discounted value of the borrower's lifetime utility under this scheme (evaluated at period 0) is

$$\delta^{T^*}\left[\sum_{t=1}^{T_M^*}(1+r)^{T^*-t}[f(k_{t-1}^*)-(1+r)k_{t-1}^*]-\bar{S}+V\right].$$

Now, we construct a new scheme $\langle\{k_t^{'}\}_{t=0}^{T_M^{'}-1},\{\alpha_t^{'}\}_{t=1}^{T_M^{'}-1},\{\beta_t^{'}\}_{t=1}^{T_M^{'}-1},T_M^{'},\{\{\beta_t^{A^{'}}\}_{t=T_M^{'}}^{T^{'}-1},T^{'}\rangle$, where

$$\begin{aligned}
&T_M^{'}=T_M^*+1,\\
&k_t^{'}=k_t^* \quad \forall t\in\{0,....,T_M^*-1\} \quad \text{and } k_{T_M^{'}-1}=k_{T_M^*-1},\\
&\alpha_t^{'}=1 \quad \forall t\in\{1,....,T_M^{'}-1\}, \quad \beta_t^{'}=0 \quad \forall t\in\{1,....,T_M^{'}-1\},\\
&\beta_t^{A^{'}}=1 \quad \forall t\in\{T_M^{'},...,T^{'}\},\\
&T^{'}=T^*.
\end{aligned}$$

First observe, $\displaystyle\sum_{t=1}^{T_M^{'}}(1+r)^{T_M^{'}-t}[f(k_{t-1}^{'})-(1+r)k_{t-1}^{'}]$

$$\begin{aligned}
&= f(k_{T_M^*-1}^*)-(1+r)k_{T_M^*-1}^*+\sum_{t=1}^{T_M^*}(1+r)^{T_M^*-t}[f(k_{t-1}^*)-(1+r)k_{t-1}^*]\\
&> \sum_{t=1}^{T_M^*}(1+r)^{T_M^*-t}[f(k_{t-1}^*)-(1+r)k_{t-1}^*],
\end{aligned}$$

where the first equality is from the construction and the second equality from the fact that $f(k_{T_M^*-1}^*)-(1+r)k_{T_M^*-1}^*>0$.

Next observe, $\forall\tau\in\{1,...,T_M^{'}\}$ $\displaystyle\delta^{T^{'}-\tau}\ \mathcal{U}^A\Big(\sum_{t=1}^{T_M^{'}}(1+r)^{T_M^{'}-t}[f(k_{t-1}^{'})-(1+r)k_{t-1}^{'}]\Big)$

$$\begin{aligned}
&> \delta^{T^*-\tau}\ \mathcal{U}^A\Big(\sum_{t=1}^{T_M^*}(1+r)^{T_M^*-t}[f(k_{t-1}^*)-(1+r)k_{t-1}^*]\Big)\\
&\geq \mathcal{U}^A(f(k_{\tau-1})),
\end{aligned}$$

where the first inequality is from the construction, as stated above, and the second inequality from the DIC constraints of the original contract.

Therefore, the borrower's lifetime utility $\delta^{T^{'}}\left[\sum_{t=1}^{T_M^{'}}(1+r)^{T_M^{'}-t}[f(k_{t-1}^{'})-(1+r)k_{t-1}^{'}]\right]$ is strictly higher under this new scheme than the original scheme, and DIC constraints are satisfied at all t under this new contract. Hence, the original scheme could not have been optimum.

The other part: $S_{T_M^*-1}^*\equiv\displaystyle\sum_{t=1}^{T_M^*-1}(1+r)^{T_M^*-1-t}[f(k_{t-1}^*)-(1+r)k_{t-1}^*]<\bar{S}$ can be shown similarly.■

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